

Uncertainty and Risk

If the probability of the occurrence of an event is '1' or '0', then that event is a certain event. On the other hand, if the probability of an event is in between 0 and 1, then that event is uncertain event. Uncertainty is also defined as randomness or stochastic nature. An uncertain event is recorded with a magnitude and probability. Data recorded as magnitude variable and its probability is defined as random variable.

Random variable recorded in a table is also known as probability distribution or probability density function.

Deviation from statistically expected value or average is uncertainty. The average of the deviations from the expected value is the measure of uncertainty. The measured uncertainty is risk. Usually, variance, the average of squared deviation, is taken as measurement of risk.

Choice under uncertain situations

Traditional theory of utility assumes that utility derived from a commodity is certain, and the consumer makes his choice by equating price and marginal utility. Moreover, the marginal utility of money is treated as constant in traditional choice theory.

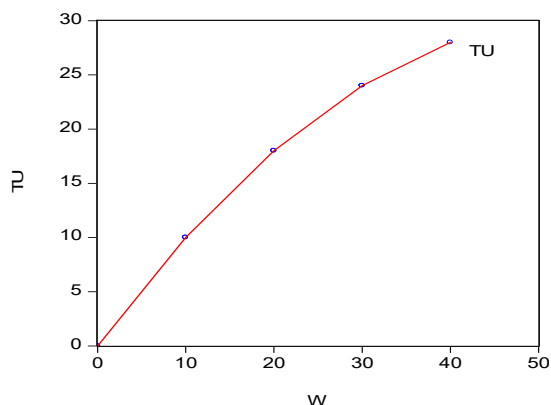
In uncertain situations, consumers make choice by equating expected utility of different random returns from a particular decision and or consumption and marginal utility of money of consumer. Consumers are classified according to the structure of the marginal utility of money. Consumers are classified under three categories, as consumers with marginal utility of money is increasing, decreasing and constant or the consumers whose total utility of money is increasing with increasing rate, increasing with decreasing rate and increasing with constant rate.

Category A

Human being with utility of money increasing with decreasing rate.

W	TU	MU
0	0	0
10	10	10
20	18	8
30	24	6
40	28	4

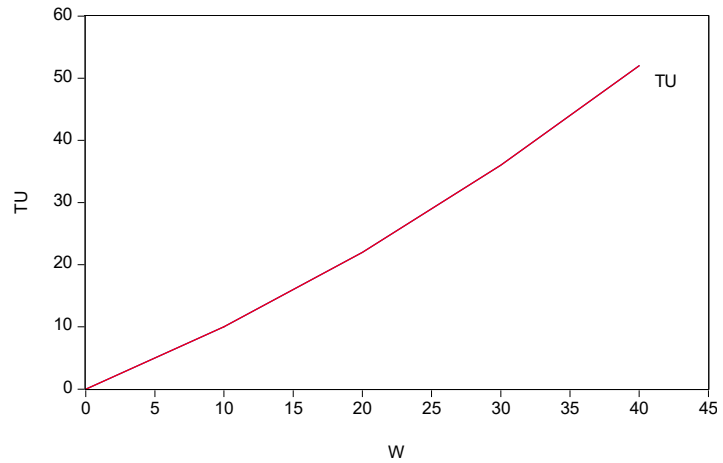
W = Wealth in terms of money
 TU = Total Utility of money
 MU = Marginal Utility of money



Category B

Human being with utility of money is increasing with increasing rate

W	TU	MU
0	0	0
10	10	10
20	22	12
30	36	14
40	52	16

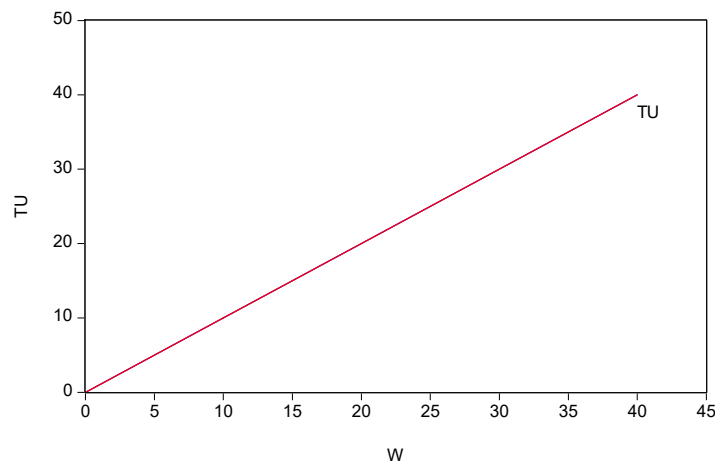


W = Wealth in terms of money
 TU = Total Utility of money
 MU = Marginal Utility of money

Category C

Human beings with utility of money is increasing with constant rate.

W	TU	MU
0	0	0
10	10	10
20	20	10
30	30	10
40	40	10



W = Wealth in terms of money
 TU = Total Utility of money
 MU = Marginal Utility of money

Decision of a person whose total utility for money is increasing at decreasing rate

Assume that the individual's wealth is zero. He has got an opportunity to invest in a share of a company. If he invests 10 in the company's share the return or the utility from that payment is not certain. He has a chance to lose 10 with 50% probably or gain additional 10 with 50% probability. Then the probability distribution or pdf is, if he loss 10, his wealth will be 10 if he gain 10, his wealth will be 30

Return	Probability	Utility
10	0.5	10
30	0.5	24
$\sum p = 1$		

His expected utility of investment of 10 is calculated as:

$$\text{Expected Utility} = E(u) = 0.5 \times (10) + 0.5 (24) = 5 + 12 = 17$$

$$\text{Expected wealth} = E(W) = 0.5 \times 10 + 0.5 \times 30 = 20$$

$$\text{Utility of expected wealth} = E(w) = u(20) = 18$$

Since his expected utility is less than the utility of the expected wealth, he will not invest. It is better for him to remain with his 20 rupees rather than to confront an uncertain situation. These type of persons are classified as risk averse consumers. Or all individuals where total utility of money is increasing at a decreasing rate are risk averse persons.

From the utility table, we can deduce that the money equivalent for 18 units of utility is 20. Then what would be the equivalent amount for 17 units of utility? Definitely that will be less than 20, and assume it as 19 and this 19 rupees is the certainty equivalent for the expected utility gained from the investment. This difference between 20 and 19 is the amount that the person will be ready to avoid risk of confronting an uncertain event and this amount is risk premium. Risk averse individuals will pay the risk premium to avoid risk. This behavior leads to the formation of insurance companies.

Decision of an individual where utility of money is increasing at an increasing rate.

W (Rs.)	TU
0	0
10	10
20	22
30	36
40	52

How does a person with total utility function of money function increasing with an increasing rate make choice under an uncertain situation explained in the example one. Let us reproduce the example.

R	P _i	U _M
10	0.5	10
30	0.5	36

Expected utility from investment:

$$E(u) = 0.5 \times (10) + 0.5 \times (36) = 5 + 18 = 23$$

$$\text{Expected wealth } E(w) = 0.5 \times 10 + 0.5 (30) = 5 + 15 = 20$$

$$\text{Utility of } E(W) = 22 \text{ (Observe the utility table)}$$

Here, utility of $E(W) <$ Expected utility from investment, i.e., $E(u)$. Then the consumer will decide to purchase the share. If a person's utility of expected wealth is less than the expected utility, then such consumers are called risk takers. This result depends on the nature of utility of money function. Invariably the table which exhibits total utility of money is increasing with an increasing rate will result in expected utility of money is greater than utility of expected income.

So all individuals with utility of money functions are increasing with increasing rate are risk lovers.

$$\text{Risk averter } u E(w) > E(u) \text{ of wealth}$$

$$\text{Risk lover } u E(w) < E(u) \text{ of wealth}$$

$$\text{Risk neutral } u E(W) = E(u) \text{ of wealth}$$

How to evaluate risk?

Results of all actions are future events. Future is uncertain in most of the situations. So individuals make decisions on the basis of their expectation about future results. Thus, expected value or mean is treated as certainty and deviation from mean 'd' or 'u' is taken is uncertainty. The mean of the deviations from mean is the measurement of uncertainty. The most appropriate measure of average of deviation is Karl Pearsons coefficient of standard deviation and so standard deviation is treated as measurement of risk in economics.

$$\mu = E(x) = \sum X_i P_i$$

$$\sigma = \sqrt{\sum (X^2 - \sum X_i P_i)^2} = \sqrt{\sum (X - \mu)^2 P_i}$$

$$\text{Coefficient of variation} = \frac{\sigma}{E(X)}$$

How does a person make his decision under an uncertain situation? How does a person's utility of money table or function influence his decision?

Let us examine these two equations by analyzing the solutions of different concrete situations.

	Investment I		Investment II	
	Return	Probability	Return	Probability
No rain	100	0.20	150	0.20
Moderate rain	200	0.50	200	0.50
Heavy rain	400	0.30	250	0.30

Investment I

$$\mu = E(x) = 0.2 (200) + 0.5 (200) + 0.3 (400) = 240$$

$$\sigma = \sqrt{0.2 (100 - 240)^2 + 0.5 (200 - 240)^2 + 0.3 (400 - 240)^2} = 111.36$$

$$C.V = \frac{111.36}{240} = 0.46$$

Investment II

$$\mu = E(x) = 0.2 (150) + 0.5 (200) + 0.3 (250) = 205$$

$$\sigma = \sqrt{0.2 (150 - 205)^2 + 0.5 (200 - 205)^2 + 0.3 (250 - 205)^2} = 35.00$$

$$C.V = \frac{35}{205} = 0.17$$

The expected return for investment one is greater than investment two. However, the risk is greater for investment one than that of two. Moreover, the risk per rupee is 0.46 in investment one and 0.17 in investment two. So investment two is less risky than investment one.